Ex 3-1

a)

$$\frac{\partial z_h}{\partial v_{h,j}} = \frac{\partial}{\partial v_{h,j}} \left(\frac{1}{1+\exp\{-\sum_{j=0}^{M} v_{h,j} x_{ij}\}} \right).$$

$$= -\frac{1}{(1+\exp\{\cdots\})^2} \frac{\partial}{\partial v_{h,j}} (1+\exp\{\cdots\})$$

$$= -\frac{1}{(1+\exp\{\cdots\})^2} \cdot \exp\{\cdots\} \cdot \frac{\partial}{\partial v_{h,j}} \left(-\sum_{j=0}^{M} v_{h,j} \cdot x_{ij} \right)$$

$$= -\frac{1}{(\quad)^2} \cdot \exp\{\cdots\} \cdot (-x_{ij})$$

$$= \frac{\exp\{\cdots\}\, x_{ij}}{(1+\exp\{\cdots\})^2} = \frac{(1+\exp\{\cdots\} - 1)\, x_{ij}}{(1+\exp\{\cdots\})^2}$$

$$= \left(\frac{1}{1+\exp\{\cdots\}} - \frac{1}{(1+\exp\{\cdots\})^2} \right) \cdot x_{ij}$$

$$= \left(1 - \frac{1}{1+\exp\{\cdots\}} \right) \cdot \frac{1}{1+\exp\{\cdots\}} \cdot x_{ij}.$$

$$= (1 - z_h)\, z_h \cdot x_{ij}.$$

b) Obviously, $z_h \in (0, 1)$. Then

$$\hat{y} = \sum_{i=0}^{M_\phi-1} w_i z_i < \sum_{i=0}^{M_\phi-1} w_i.$$

$$\hat{y} = \sum_{i=0}^{M_\phi-1} w_i z_i = w_0 + \sum_{i=1}^{M_\phi-1} w_i z_i > w_0 + \sum_{i=1}^{M_\phi-1} w_i \cdot 0 = w_0.$$

Thus. $\hat{y} \in \left(w_0, \sum_{i=0}^{M_\phi-1} w_i \right).$

c)

i) $z_h = \frac{1}{1+\exp\{0\}} = 0.5$ $\hat{y} = \sum_{i=0}^{M_\phi - 1} w_i z_i = 0.5 \sum_{i=1} w_i + w_0$.

ii) $z_h = \frac{1}{1+\exp\{-\sum_{j=0}^{M} c x_{ij}\}}$. $\hat{y} = \sum_{h=0}^{M_\phi-1} w_h z_h = w_0 + \sum_{h=1}^{M_\phi - 1} w_h z_h$.

$$= \sum_{h=0}^{M_\phi-1} \left[w_h \cdot \frac{1}{1+\exp\{-\sum_{j=0}^{M} c x_{ij}\}} \right] + w_0$$

$$= \underset{\text{offset.}}{w_0} + \underset{\substack{\text{scaling} \\ \text{factor of wights.}}}{\sum_{h=0}^{M_\phi-1} w_h} \left(\underset{\text{constant of } (x)}{\frac{1}{1+\exp\{\dots\}}} \right).$$

Layer 1

0 0 1 1 0 0 -1 -1 0 0
0 0 -1 -1 0 0 1 1 0 0

↓

0 1 0 -1 0
0 -1 0 1 0

↓

0 -4 0

↓

~~4~~ ~~-4~~

↓

sig(4) sig(-4)

0.99 0.01

0 0 -1 -1 0 0 1 1 0 0
0 0 1 1 0 0 -1 -1 0 0

↓

0 -1 0 1 0
0 1 0 -1 0

↓

0 4 0

-4 4

sig(4) sig(-4)

0.99 0.01