Ex 6-1

X	0	1	2	3
$P(X)$	$2\theta/3$	$\theta/3$	$2(1-\theta)/3$	$(1-\theta)/3$

3, 0, 2, 1, 3, 2, 1, 0, 2, 1

$$L(\theta) = \left[\frac{(1-\theta)}{3}\right]^2 \left[\frac{2\theta}{3}\right]^2 \left[\frac{\theta}{3}\right]^3 \left[\frac{2(1-\theta)}{3}\right]^3$$

$$\log L(\theta) = \log\left(\frac{1}{3^2}\cdot\frac{2^2}{3^2}\cdot\frac{1}{3^3}\cdot\frac{2^3}{3^3}\right) + 2\log(1-\theta) + 2\log\theta$$
$$+ 3\log\theta + 3\log(1-\theta)$$

$$= \text{constant} + 5[\log(1-\theta) + \log\theta]$$

$$\frac{\partial \log L(\theta)}{\partial\theta} = 5\left[-\frac{1}{1-\theta} + \frac{1}{\theta}\right] = 0$$

If $\theta \neq 1$ and $\theta \neq 0$ we have:

$$-\theta + 1 - \theta = 0 \Rightarrow \theta = \frac{1}{2}$$

$$\frac{\partial^2 \log L(\theta)}{(\partial\theta)^2} = -\frac{1}{(1-\theta)^2} - \frac{1}{\theta^2} < 0$$

~~convex~~ concave

Ex 6-2

a) $p(x_i) = p(1-p)^{x_i}$

$$\log L(p) = \sum_{i=1}^{N} \log\left[p(1-p)^{x_i}\right]$$

$$= \sum_{i=1}^{N} \left[\log p + x_i \log(1-p)\right]$$

$$\frac{\partial \log L(p)}{\partial p} = \sum_{i=1}^{N} \left[\frac{1}{p} - x_i \cdot \frac{1}{1-p}\right] = 0$$

$$= \frac{N}{p} - \frac{1}{1-p} \cdot \sum_{i=1}^{N} x_i = 0$$

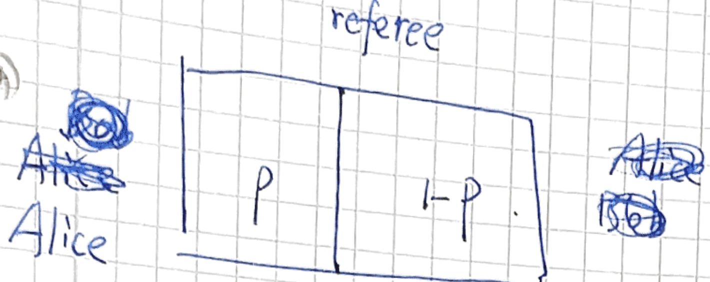
$$\frac{1-p}{p} = \left(\frac{1}{N}\sum_{i=1}^{N} x_i\right) \cdot p + p$$

$$\frac{1}{1+\frac{1}{N}\sum_{i=1}^{N} x_i} = p$$

b) $X = \{7, 2\}$ $x_1 = 7$, $x_2 = 2$ $N=2$

$$\hat{p}_{MLE} = \frac{1}{1+\frac{1}{2}(7+2)} = \frac{2}{2+9} = \frac{2}{11}$$

Ex 6-3



Alice 5 points.

Bob 3 points.

a)

$$P(\text{Bob win}) = (1-p)^3 = \left(1-\frac{2}{3}\right)^3 = \frac{1}{3^3} = \frac{1}{27}$$

b)

$$L(p) = (1-p)^3 p^5$$

X	1-p	p
p(X)		

$$\log L(p) = 3\log(1-p) + 5\log p$$

$$\frac{\partial L(p)}{\partial p} = 3\frac{-1}{1-p} + 5\frac{1}{p} = -3p + 5 - 5p = 0$$

$$\Rightarrow \underline{p = \frac{5}{8}} \quad \hat{p}_{MLE}$$

$$p(\text{Bob win}) = (1-p)^3 = \left(1-\frac{5}{8}\right)^3 = \left(\frac{3}{8}\right)^3 \approx 0.053$$

~~E(Bob win)~~

$$\underline{P(p \mid A=5, B=3)} \qquad \mathbb{E}(\text{Bob win}) = \int_0^1 (1-p)^3 P(p \mid A=5, B=3)\, dp$$

$$= \frac{P(A=5, B=3 \mid p)\, P(p)}{\int_0^1 P(A=5, B=3 \mid p)\, P(p)\, dp}$$

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