Ex 9-7

Lagrange Mutipliers

$f = 0$

condition $g = 0$

minimum

$\nabla f = \lambda \nabla g$

$$\begin{cases} \frac{\partial f}{x_i} + \lambda \frac{\partial g}{y_i} = 0 \\ g = 0 \end{cases}$$

a) $f(x,y,z) = \cancel{x^2+y^2} \; x+y+2z$

$g(x,y,z) = x^2+y^2+z^2-3$

$\frac{\partial f}{\partial x} = 1 \quad \frac{\partial f}{\partial y} = 1 \quad \frac{\partial f}{\partial z} = 2$

$\frac{\partial g}{\partial x} = 2x \quad \frac{\partial g}{\partial y} = 2y \quad \frac{\partial g}{\partial z} = 2z$

$$\begin{cases} 1-2\lambda x = 0 \\ 1-2\lambda y = 0 \\ 2-2\lambda z = 0 \end{cases}$$

$$\Rightarrow \begin{cases} x = \frac{1}{2\lambda} \\ y = \frac{1}{2\lambda} \\ z = \frac{1}{\lambda} \end{cases}$$

$\cancel{\Rightarrow f = \frac{1}{2\lambda} + \frac{1}{2\lambda} + \frac{2}{\lambda} =}$

$g \Rightarrow \frac{1}{4\lambda^2} + \frac{1}{4\lambda^2} + \frac{4}{4\lambda^2} = 3$

$\frac{3}{2\lambda^2} = 3 \Rightarrow \lambda = \pm\frac{\sqrt{2}}{2}$

$$\Rightarrow \begin{cases} x_1 = \sqrt{2}/2 \\ y = \sqrt{2}/2 \\ z_1 = \sqrt{2} \end{cases} \quad \text{or} \quad \begin{cases} x_2 = -\sqrt{2}/2 \\ y_2 = -\sqrt{2}/2 \\ z_2 = -\sqrt{2} \end{cases}$$

$f(x_1,y_1,z_1) = \sqrt{2} + 2\sqrt{2} = 3\sqrt{2}$ maximum

$f(x_2,y_2,z_2) = -\sqrt{2} - 2\sqrt{2} = -3\sqrt{2}$ minimum

b) $f(x,y) = xy$

$g(x,y) = 3x^2+y^2-6$

$3x^2 \cancel{=6}-y^2$

$y^2 = 6-3x^2$

$\cancel{y = (\sqrt{6-3x^2})}$

$\frac{\partial f}{\partial x} = y \quad \frac{\partial f}{\partial y} = x \quad \frac{\partial g(x,y)}{\partial x} = 6x \quad \frac{\partial g}{\partial y} = 2y$

$y - 6x\lambda = 0 \quad \Rightarrow \quad y = 6x\lambda = 6\lambda \cdot 2\lambda \cdot y$

$x - 2y\lambda = 0$

$\Rightarrow \quad y = 12\lambda^2 y$

if $y=0$ then $x=0$ contradict with $3x^2+y^2=6$

thus $y \neq 0 \quad \Rightarrow \quad \lambda = \pm\frac{1}{\sqrt{12}} \quad (\lambda^2 = \frac{1}{12})$

$3 \cancel{x^2 + \Rightarrow x =}$

$3x^2 + (6x\lambda)^2 - 6 = 0$

$3x^2 + 36x^2\lambda^2 - 6 = 0$

$\Rightarrow 3x^2 + \frac{36}{12}x^2 - 6 = 0$

$\Rightarrow 6x^2 = 6 \Rightarrow x = \pm 1$

$\Rightarrow y^2 = 36x^2\lambda^2 = 3x^2 = 3$

$y = \pm\sqrt{3}$

$P_1 = (1, \sqrt{3})$

$P_2 = (1, -\sqrt{3})$

$P_3 = (-1, \sqrt{3})$

$P_4 = (-1, -\sqrt{3})$

$g_1 = 3x^2 + y^2 - 6 = 3 \times 1 + 3 - 6 = 0$

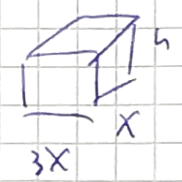
$g = 0$

$P_1 \; P_2 \; P_3 \; P_4 \Rightarrow g = 0$

maximum

$g(1,1) = 3+1-6 = -2 < g$

Ex 9-2



$$f(x,h) = 2(3x^2+3xh+xh) = 6x^2+8xh$$

$$g(x,h) = 3x^2h - 36$$

$$\frac{\partial f}{\partial x} - \lambda \frac{\partial g}{\partial x} = 12x + 8h - 6xh\lambda = 0$$

$$\frac{\partial f}{\partial h} - \lambda \frac{\partial g}{\partial h} = 8x - 3x^2\lambda = 0 \Rightarrow \lambda = \frac{8}{3x}$$

$$\Rightarrow 12x + 8h - \overset{2}{\cancel{6}}xh \cdot \frac{8}{\cancel{3x}} = 12x + 8h - 16h = 12x - 8h = 0$$

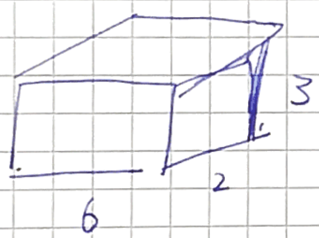
$$\Rightarrow 3x - 2h = 0$$

$$h = \frac{3x}{2}$$

$$3x^2h - 36 = 3x^2 \cdot \frac{3x}{2} = 36$$

$$\Rightarrow x^3 = 8 \Rightarrow x = 2$$

$$\Rightarrow h = \frac{3 \times 2}{2} = 3$$



$$f(x,h) = 6 \times 4 + 8 \times 2 \times 3$$

$$= 24 + 48$$

$$= 72 \text{ cm}^2$$

max

$\cancel{f(1,1) = 6 + 8 = 14 \text{ cm}^2}$

$$3x^2h = 36$$

$\left.\begin{array}{l}\cancel{x=2} \\ h=3\end{array}\right\} \Rightarrow g = 36$

$\cancel{f = 6 \times 4 + 8 \times 2 \times}$

min

$$\begin{cases} x = 1 \\ h = 12 \end{cases}$$

$$f = 6 + 8 \times 12 = 96 + 6 = 102$$

Ex 9-3

input x

output $h(x)$

let's consider $D = \{ x_1, x_2, \cdots, x_n \}$

fold : $D \setminus \{x_i\}$ have two case :

parity of $D \setminus \{x_1\}$ is 1 $\Rightarrow h(x_1) = 1$

parity of $D \setminus \{x_1\}$ is 0 $\Rightarrow h(x_1) = 0$

$p(h(x_1) = 1) = \frac{1}{2} = p(h(x_1) = 0)$

$\Rightarrow \mathbb{E}($ ~~parity~~ fold of $D \setminus \{x_1\})$ is $\frac{1}{2} \times 0 + \frac{1}{2} \times 1 = \frac{1}{2}$

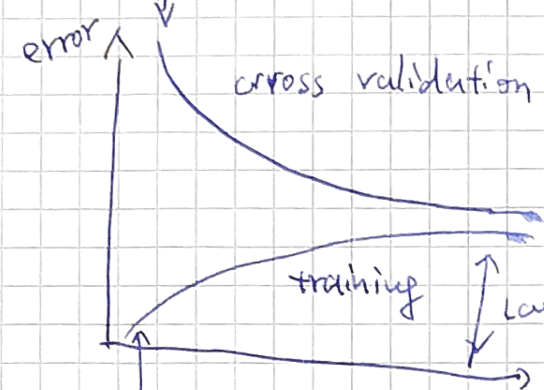
Same $\mathbb{E}($ fold of $D \setminus \{x_i\})$ is $\frac{1}{2}$

the $\mathbb{E}($ Leave one out $) = \frac{1}{2} \times n \times \frac{1}{n} = \frac{1}{2}$

Ex 9-4

too less data, easy to overfit. traingin samples

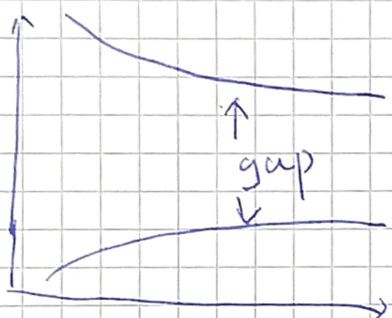
a)



large e ← ~~high~~ hypothsis space not sutable.

easy to fit.

High bias   High variance

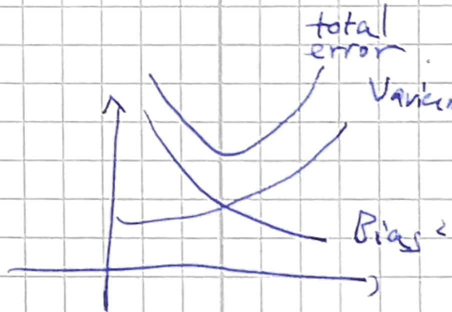


model too expressive, overfit data.

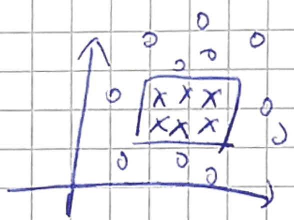
but will converge.

b) Bias – Variance Trade off.

$\mathbb{E}(\text{Loss}) = (\text{Bias})^2 + \text{variance}$.



i)



case i: model complexity small
variance small.

case ii: mode complexity large.
bias small

If limit #samples the case 1.