Ex 10-1.

Assignment due $A=a$. $A=\{a, \neg a\}$

News group is busy $B=b$ $B=\{b, \neg b\}$

Computer lab is full $C=c$. $C=\{c, \neg c\}$

B, C conditionally independent.

$\Rightarrow P(BC|A) = P(B|A)\,P(C|A)$

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1)

$P(A,B,C)$			
a	b	c	0.032
a	b	$\neg c$	0.048
a	$\neg b$	c	0.008
a	$\neg b$	$\neg c$	0.012
$\neg a$	b	c	0.054
$\neg a$	b	$\neg c$	0.216
$\neg a$	$\neg b$	c	0.126
$\neg a$	$\neg b$	$\neg c$	0.504

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2)

$P(B,C)$		
b	c	0.086
b	$\neg c$	0.264
$\neg b$	c	0.134
$\neg b$	$\neg c$	0.516

Notice:

$P(B=b) = 0.35$

$P(C=c) = 0.22$.

$P(B=b, C=c) = 0.086$

$P(B=b)\,P(C=c) = 0.35 \times 0.22 \neq 0.086$

$= 0.077$

3) $P(A \mid B=b)$

$$\Rightarrow P(A=a \mid B=b) = \frac{P(A=a, B=b)}{P(B=b)} = \frac{0.08}{0.35} = 0.229$$

$$\Rightarrow P(A=\neg a \mid B=b) = 1 - 0.229 = 0.771$$

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4) $P(A \mid B=b, C=c)$

$$\Rightarrow P(A=a \mid B=b, C=c) = \frac{P(A=a, B=b, C=c)}{P(B=b, C=c)} = \frac{0.054}{0.086} = 0.3/2$$

$$\Rightarrow P(A=\neg a \mid B=b, C=c) = 1 - 0.3/2 = 0.628$$

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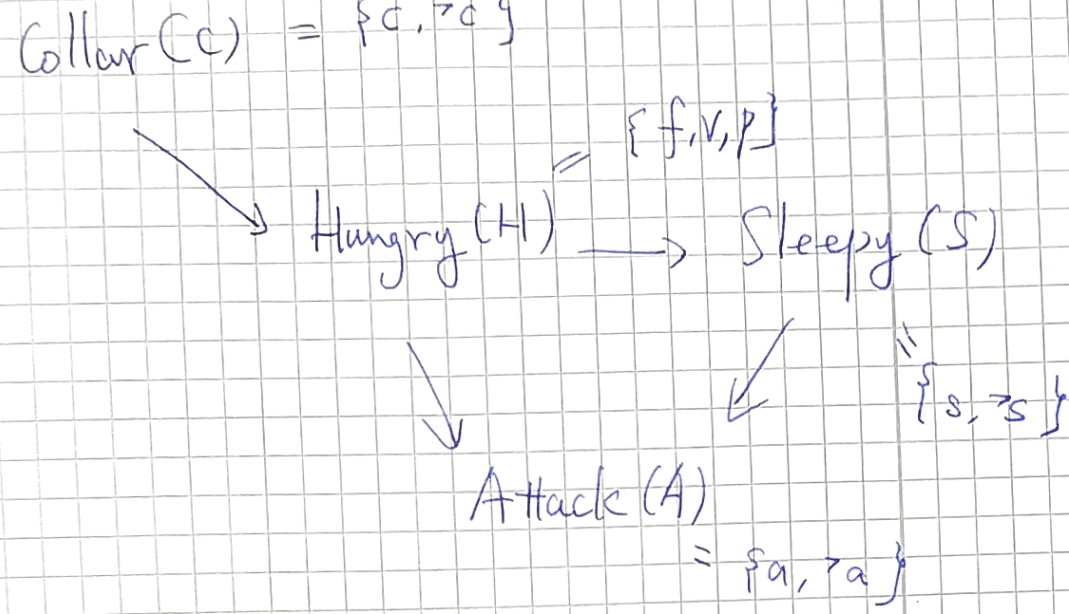
5) Observation:

$$P(A=a) < P(A=a \mid B=b) < P(A=a \mid B=b, C=c)$$

more evidence increases belief in $A=a$

Ex 10-2

a)



b) $P(A=a, C=c, S=s, H=f)$

$= P(A=a \mid H=f, S=s) \cdot P(S=s \mid H=f) \cdot P(H=f \mid C=c) P(C=c)$

$= 0.01 \times 0.9 \times 0.7 \times 0.4 = 0.00252$

$P(A=a, C=c, S=s)$

$= P(A=a, C=c, S=s, H=f) + P(A=a, C=c, S=s, H=V) + P(A=a, C=c, S=s, H=P)$

$= 0.01692$

c) $P(C=c, S=s) = P(C=c, S=s, H=f) + P(C=c, S=s, H=V) + P(C=c, S=s, H=P)$

$= 0.3$

$P(A=a \mid C=c, S=s) = \dfrac{P(A=a, C=c, S=s)}{P(C=c, S=s)} = 0.0564$

c)

$$\mathbb{E}(\text{utility}) = -10 \times P(A=a \mid C=c, S=s) + 5 \times P(A=\neg a \mid C=c, S=s)$$

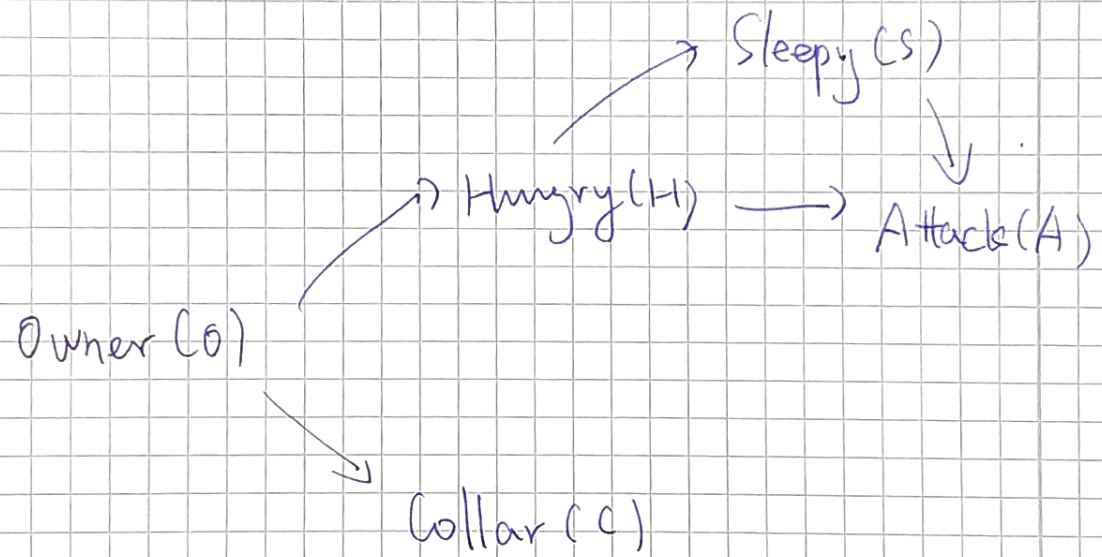
$$= 4.154$$

$$\mathbb{E}(\text{utility}) = x\, P(A=a \mid C=c, S=s) + 5 P(A=\neg a \mid C=c, S=s)$$

$$< -2$$

$$\Rightarrow x < -119.113$$

d)



Ex 10-3

$m_k = \max\{m_{k-1}, \cdots, m_1\}$.

a) Yes. $P(X_k \mid X_{k-1}, \cdots, X_1) = P(X_k \mid X_{k-1})$

b) Yes. $P(X_k \mid X_{k-1} \cdots X_1) = P(X_k \mid X_{k-1})$

c) Y_k = result (k^{th}) $\quad X_1 = Y_1 \quad X_k = Y_k + Y_{k-1}$.

$Y_k = \{1, 2, 3, \cdots, 6\}$

$X_{k-1} = Y_{k-1} + Y_{k-2}$

$\vdots$

$X_2 = Y_2 + Y_1$

$\Rightarrow X_k = X_{k-1} - X_{k-2} - \cdots - X_2 + X_1 + Y_k$.

$\Rightarrow X_k = f(X_{k-1}, \cdots, X_1) + Y_k$.

which is not a markov chain.

d) $Y_k = 1$ if k^{th} result is odd.

$Y_k = 0$ otherwise (even)

$X_k = Y_k X_{k-1}$ typical markov process.